Reliability is a Fungible Resource

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Fungible Resources

- $$ ⇔ Gold
- Euros ⇔ $$
- Mass ⇔ Energy
- Space ⇔ Time ⇔ Energy

- Claim:

Reliability ⇔ Space / Time / Energy

Reliability and Area (I)

- But also $P_{\text{fail}}(A)$

- Example: MOSFET
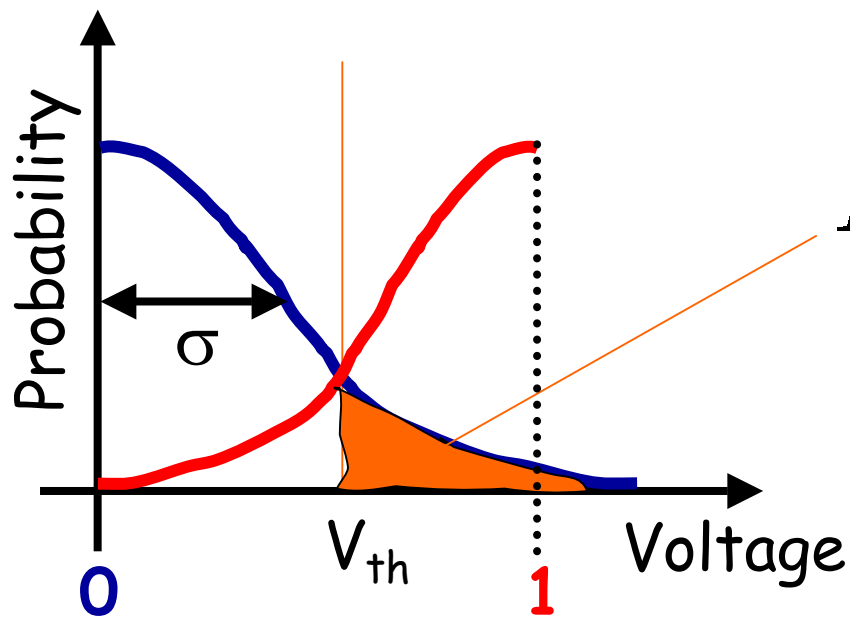
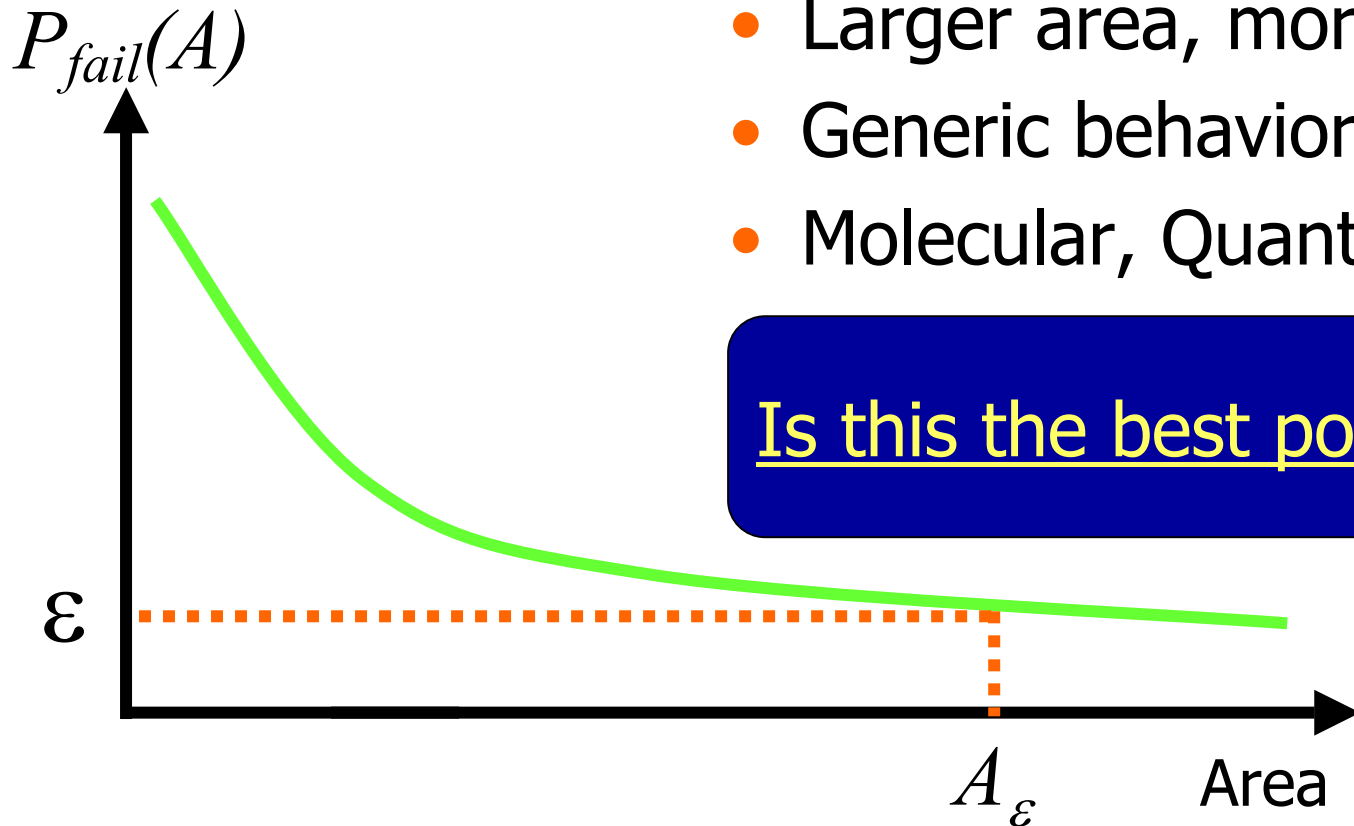$$P_{\text{fail}} = P[V > V_{th}]$$

$$= \text{erfc}(V/\sigma)$$

$$= \text{erfc}(A/A_0)$$

$$\sigma \sim \frac{dV}{dQ} \sim \frac{1}{C} \sim \frac{1}{A}$$

$P_{fail} \leftrightarrow$ Area Fungibility

- What tradeoffs are possible?

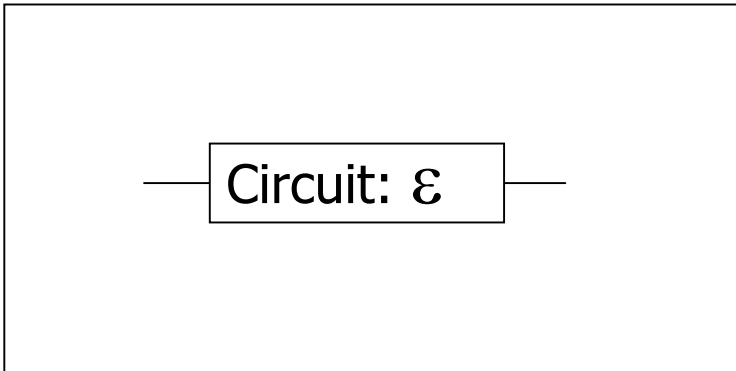$P_{fail}(A)$

- Larger area, more reliable
- Generic behavior!
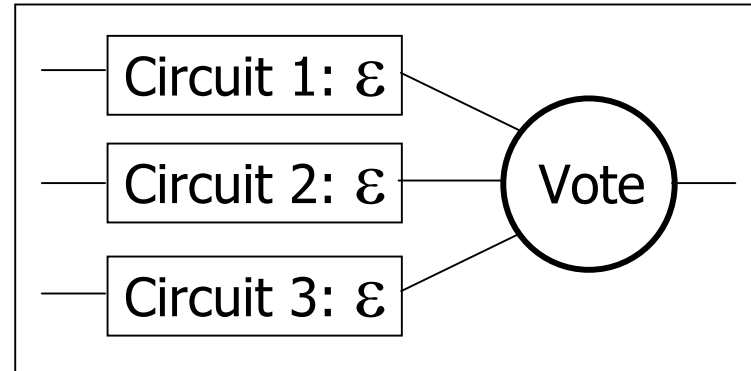- Molecular, Quantum, …

Is this the best possible?

ε

A_ε Area

Reliability and Area (II)

- Consider a circuit which fails with probability ε

Area: A
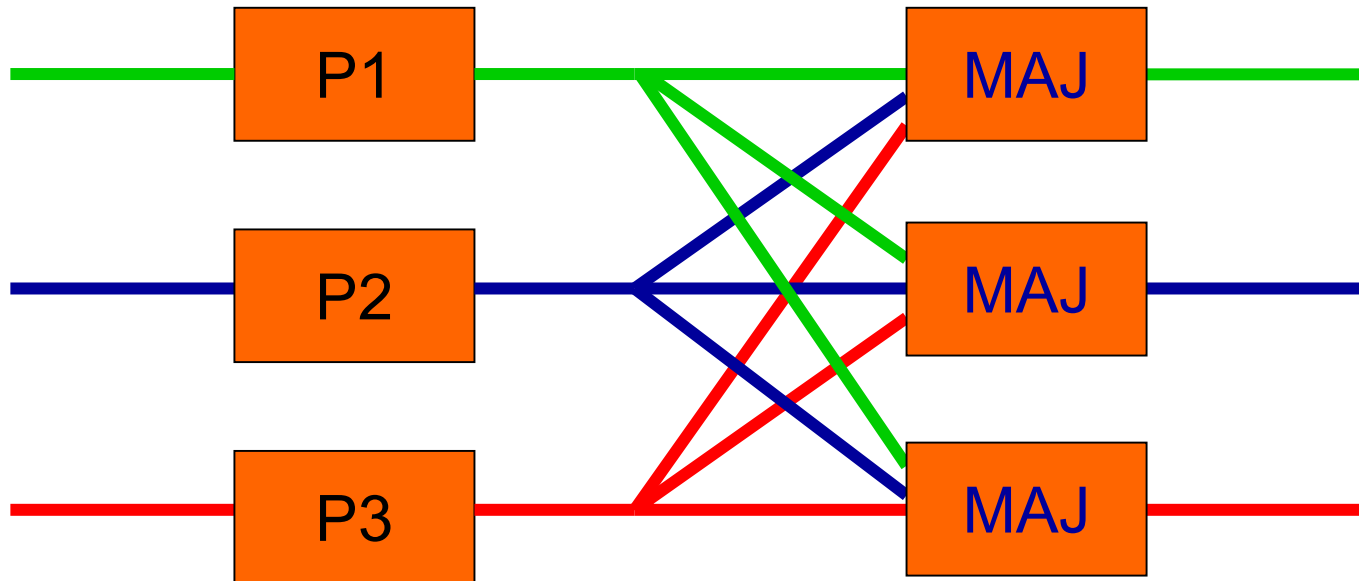
P_{fail}: ε

Area: ~3A
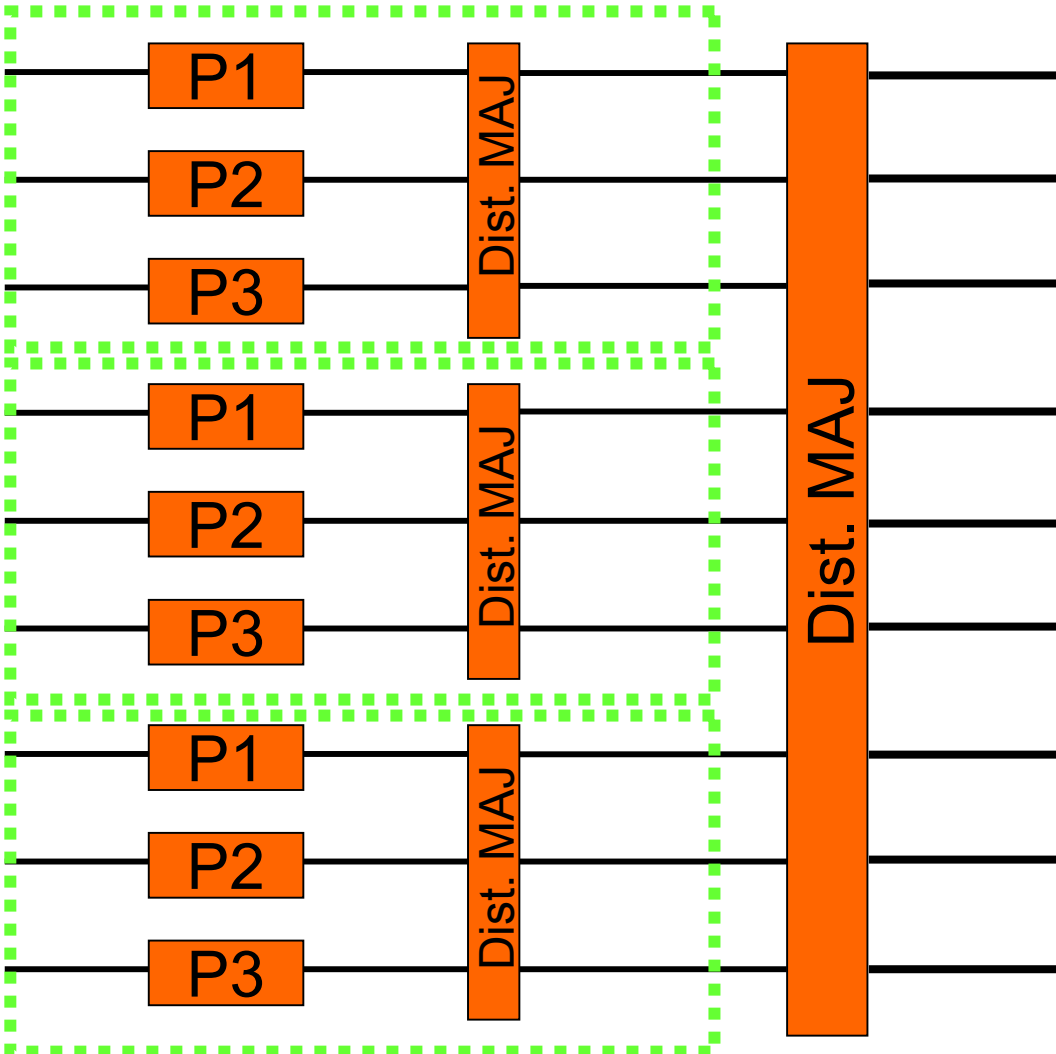
P_{fail}:   ~$3\varepsilon^2$

Distributed Voting

- Immune to any single gate failure

- Failure probability: $\leq (cP)^2$

Double Encoding

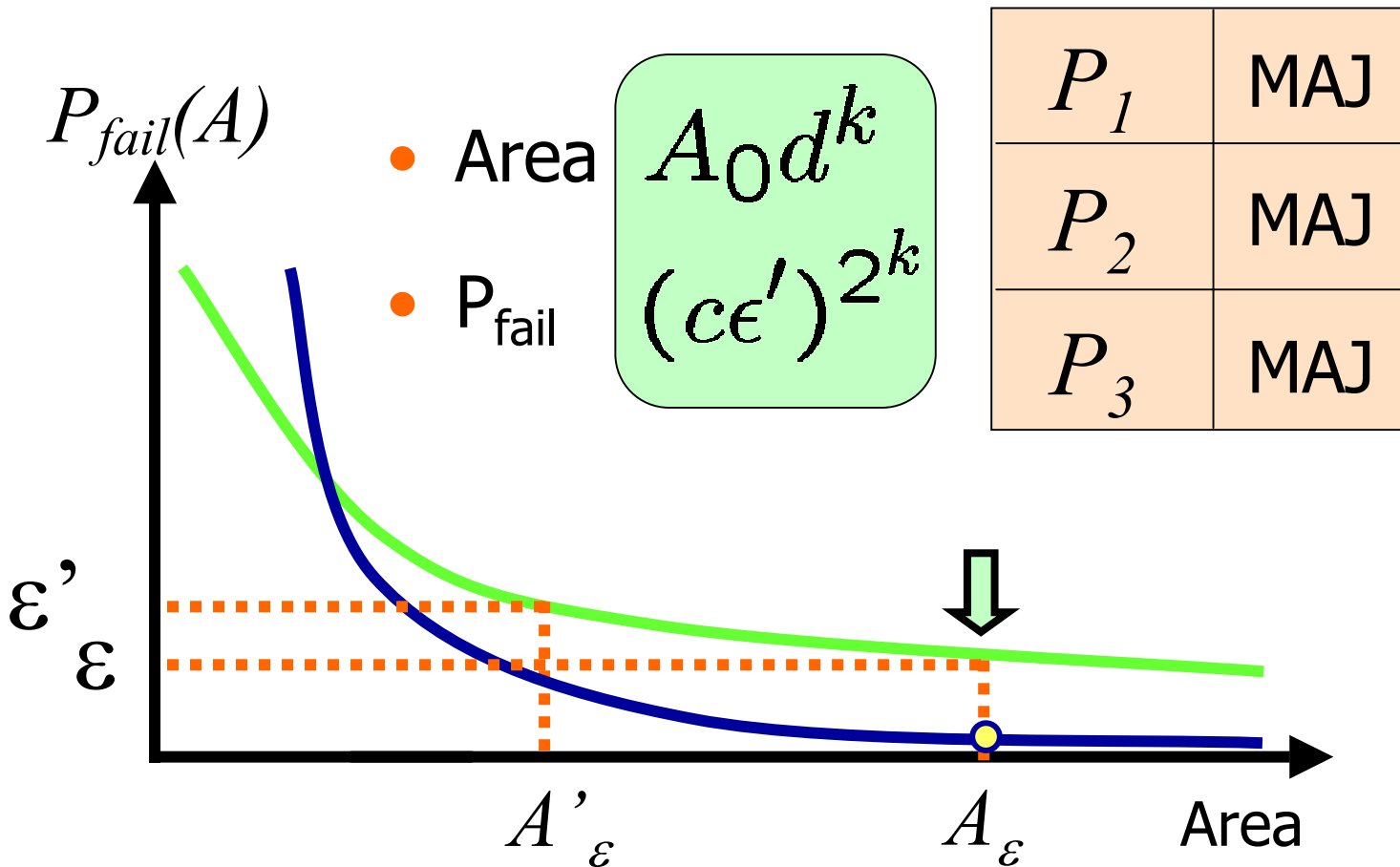Area: 3×3^2
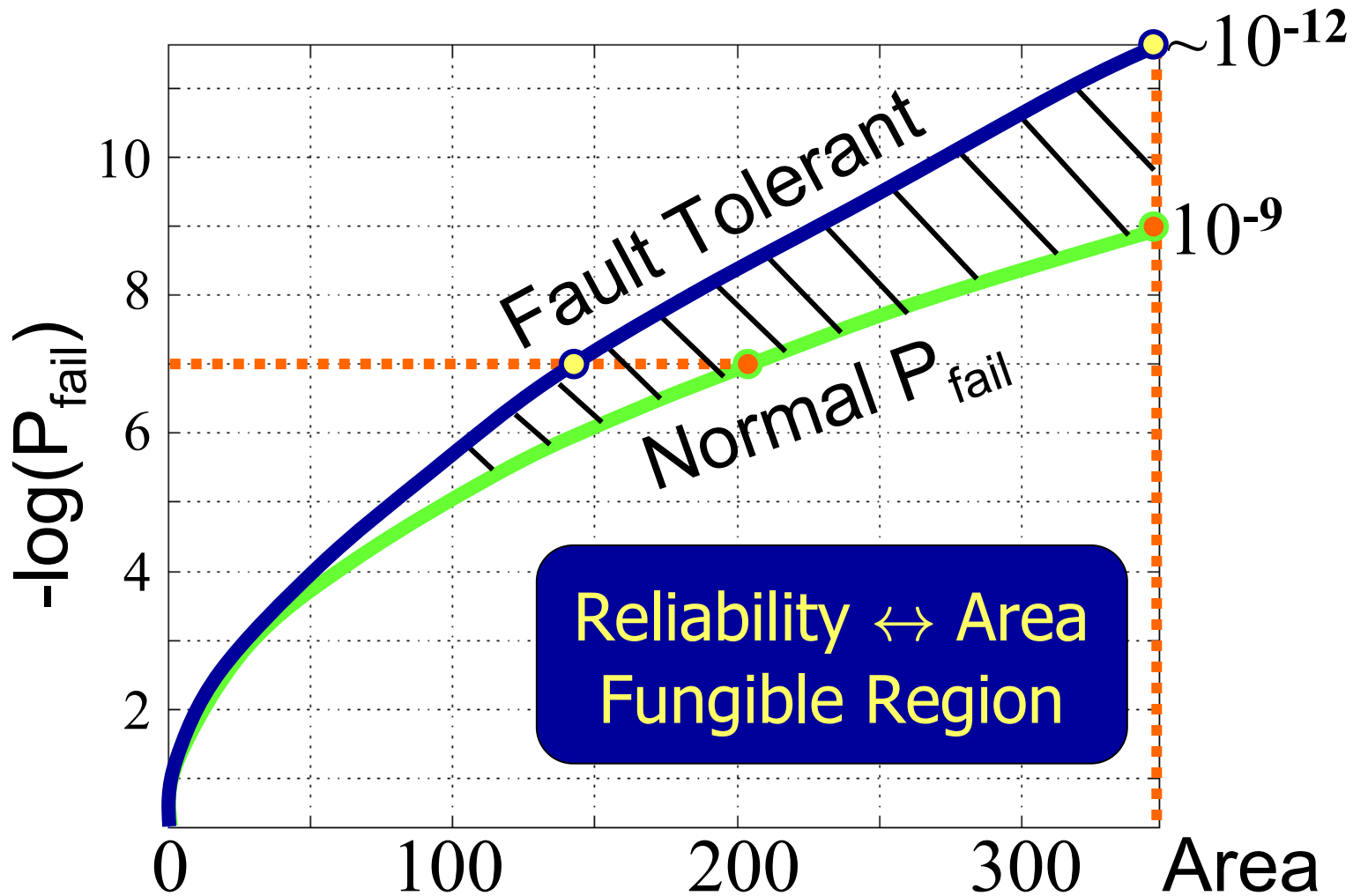
Failure probability:

$$F \leq (cP)^4$$

Recursive Encoding

- Recursion: k

- Area: $A_0 d^k$

- Failure probability: $F \leq (cP)^{2^k}$

- Threshold: $P \leq 1/c$

P$_{\text{fail}}$ ↔ Area Fungibility

- What tradeoffs are possible?

$$P_{fail}(A)$$

- Area $A_0 d^k$

- P$_{\text{fail}}$ $(c\epsilon')^{2^k}$

P_1	MAJ
P_2	MAJ
P_3	MAJ

ε'

ε

A'_ε A_ε Area

Example: erfc(A$^{1/4}$)

Only for some functions

Function	Effective Region
$p(A) = \dfrac{1}{A^\gamma}$	$\gamma \leq 5$
$p(A) = \exp(-A^\gamma)$	$\gamma \leq 0.48$
$p(A) = \operatorname{erfc}(A^\gamma)$	$\gamma \leq 0.27$

Conclusions

Reliability $\leftrightarrow$ Space, Time, Energy

Fault Tolerance can save resrouces

- CMOS?

$$P_{fail} = \mathrm{erfc}\left(1/\sqrt{\frac{K'_w(\gamma)}{v_{th}^2}\frac{1}{TP\gamma} + \frac{K'_f}{v_{th}^2}\frac{1}{TA}}\right)$$

- Neurons